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Teachers' Misconception on Fractions Based on the Errors of Elementary Students

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Abstract

This study used qualitative descriptive research to analyze the factors leading to teachers' misconceptions about fractions based on students' errors. The results revealed that the teachers experienced misconceptions in ordering fractions, illustrating fractional numbers whose numerator is greater than its denominator, dividing numbers with zero, and understanding word or story problems related to fractions. The cause of the teachers' misconception in ordering fractions was their misunderstanding of the questions. The cause of the teachers' misconception in illustrating fractional numbers that the numerator is greater than its denominator was their misunderstanding of the fractions. Furthermore, the cause of the teachers' misconception in dividing numbers by zero was their assumption that the lower the denominator, the greater its results. The last, the cause of the teachers' misconception in understanding word problems related to fractions was they misunderstood the information given in the questions.

Keywords: Mathematical misconceptions, Errors, Fractions.

1. Introduction

Adding, subtracting, multiplying and dividing fraction numbers are mathematical competencies taught in elementary school¹. Students at elementary schools are expected to know how to read fractional numbers and solve problems related to fractions. The primary purpose of teaching and learning fractions is to help students constructing a basic idea of how to divide the equal share as part of a whole. Therefore, the concept of fractions becomes a pre-requisite understanding to be able to study mathematics at the next level².

Results of studies show that students' difficulties in learning fractions were due to the obstacles of understanding the concept of fractions such as understanding equivalent fractions, comparing two fractions with different denominators, and operating two fractions^{3,4}. The facts mentioned that 39.1%

of students added the numerator to the numerator, the denominator to the denominator, as well as in the reduction of fractions⁵.

One of the factors causing students' difficulties in understanding fractions is teachers' complicated explanation in the classroom. Thus students have difficulties in connecting the concepts they receive. Besides, teachers perform quick explanation and consider the topic is uncomplicated lead teachers to not profoundly explain the topic, as students can read the full explanation in the textbook for themselves. The results show that prospective teachers often ignore the difficulties experienced by students. This fact occurs because prospective teachers have limited understanding skills in identifying students' difficulties⁶.

The ability to understand mathematical concepts should be possessed by every prospective teacher, especially experienced teachers. Since mathematical concepts are interrelated each other, mastering the concepts is crucial⁷. For example, in teaching fractions, teachers must perform apperception of integers and number lines concepts. Therefore, teachers who are not able to associate the concepts will experience misconceptions.

Misconceptions are concepts not in accordance with scientific meanings or experts' understanding in a particular field. Misconceptions refer to misunderstanding and misinterpretation based on the incorrect meaning⁹. Misconceptions arise because of insufficient knowledge conceived by someone before hand, which is in the form of an incomplete basic concepts¹⁰.

The teachers experienced misconceptions in comparing $\frac{2}{3}$ and $\frac{3}{4}$. The teachers perceived the greater the denominator, the greater the fraction result¹¹. The teacher answered correctly was only 32.50%, but their reason was inaccurate. In addition, misconceptions experienced by prospective teachers included factors in the language usage, including distinguishing expressions such as *of a*, *of one*, *of the*, and *of each*¹². Furthermore, the result¹³ shows that prospective teacher education students experienced misconceptions in the concepts of fractions, prime numbers, integers, mixed operations, and operations on fractions.

Based on the statements above, a study of teachers' misconception is carried out. This study aims to analyze the causes of misconceptions experienced by teachers based on the errors of elementary students.

2. Review of Literature

2.1. Misconceptions

Misconceptions refer to one of the concepts that are different from its scientific meaning proposed by experts in certain fields⁸. Misconceptions can also be interpreted as a false belief that is not in accordance with the generally accepted explanation and proven valid statements about a phenomenon or event¹⁴. Misconceptions are misunderstanding and misinterpretation based on incorrect meaning¹⁵.

Misconceptions occur when someone believes in an objectively wrong concept. If a concept cannot be proven right or wrong, someone cannot be judged as experiencing misconceptions about the concepts¹⁶. Misconceptions occur repeatedly and explicitly so that they can cause learning difficulties when students apply the concepts to interpret new experiences^{17,18}. Therefore, this misunderstanding is often perceived as maintained by students, even though the teacher has tried to provide an opposite reality¹⁹.

Misconceptions differ from errors as errors refer to the wrong answer because improper and unsystematic planning applied in solving mathematical problems, while misconceptions are symptoms of cognitive structures that cause errors²⁰. The idea of misconceptions refers to the views causing a series of errors resulted from the premise errors underlying a particular concept or process²¹. Therefore, misconceptions are one of the error sources in mathematics, beside others such as carelessness or the use of ambiguous language. Misconceptions of prior knowledge will hinder the process of understanding new knowledge engendering errors in learning mathematics.

2.2. Factors contributing to Misconceptions

Students' misconceptions can occur as a result of observation towards natural phenomena around them; for instance, students' feelings sometimes can refute them in understanding the phenomenon. Misconceptions can also emerge because of concepts taught are inaccessible by students' mental development, which means the information comes from outside and inside the classroom is a potential source of misconception if the information captured by students does not rectify the students' mental picture²².

Students' initial conceptions before attending school are very influential in understanding the following concepts. When the initial conceptions possessed by students contain misconceptions, they will cause misconceptions in the following subjects. This is because the wrong interpretation is based on students' personal views. Therefore, students must understand the relationship among concepts, that the concepts are interdependent as each concept relates to others.

Specific causes that might lead students to experience misconceptions originating from teachers are teachers do not master the topic proficiently, completely, and correctly. In addition, teachers do not have an educational background in the field taught. Another fact is teachers rarely carry out learning activities that give students the opportunity to apply their ideas; accordingly, they cannot overcome misconceptions early. The cause of students' misconceptions is also because there was no positive relationship established between teachers and students; hence students are reluctant to ask questions when they experience difficulties in understanding.

3. Methodology

The qualitative descriptive research method was applied in this study. The subjects were selected based on the students' conditions. Six students who made the most errors and were able to communicate well were chosen as research subjects. Meanwhile, two teachers teaching in grades fourth and fifth were involved in this study.

The data were collected using diagnostic tests, interviews, and documentation. Interviews were conducted to clarify each comment given by the subjects to the answers of the students, while the documentation used in this study was in the form of students' answer sheets, notebooks, exercises, and others. This research used data source and data collection technique triangulation.

4. Results

Data about misconceptions encountered by the teachers were obtained based on errors experienced by the students on diagnostic tests and unstructured interviews. Based on the students' answers to the diagnostic tests performed, four errors were found, namely error in ordering fractions, error in comparing two fractions, error in fraction operations, and error in understanding the story questions related to fractions. Two of the four students' errors were caused by the misconceptions experienced by the teachers, i.e., misconceptions in comparing two fractions and misconceptions in understanding story problems. In addition, teachers experienced misconceptions in ordering fractions and dividing numbers by zero.

The students' errors in ordering fractions were their apprehension of the order of fractions that are equal to the order of integers. The students ordered $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ because after 3 is 4, then 5. This is an error. However, this perception was also found in the teachers who taught mathematics, but the reasons stated were different. The teacher did not find numbers between $\frac{1}{3}$ and $\frac{1}{5}$. According to her, there was no exact number between the two numbers. For that matter, the teacher should be able to determine the equivalent numbers from $\frac{1}{3}$ and $\frac{1}{5}$, for example $\frac{1}{3} = \frac{5}{15}$ and $\frac{1}{5} = \frac{3}{15}$, so the number between $\frac{5}{15}$ and $\frac{3}{15}$ is $\frac{4}{15}$. The teacher's misconceptions in ordering fractions can be seen in figure 1 below.

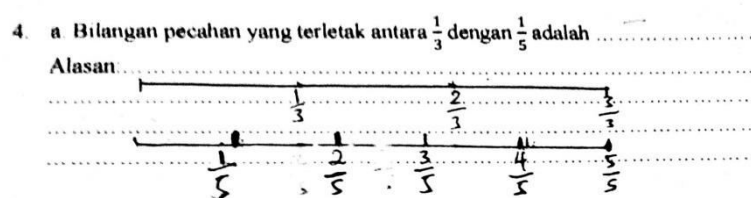


Figure 1: The teachers' misconceptions in ordering fractions

The following is a snippet of interviews between the researcher and the teachers.

Researcher : *Why don't you provide an answer for question number 4?*

The teacher : *The question asks for numbers between $\frac{1}{3}$ and $\frac{1}{5}$. I make a number line. However, there is no number between those two numbers in the number line. That means there is no number between those numbers.*

Based on the snippet of interviews above, the teacher experienced misconceptions in ordering fractions using a number line. The misconceptions were caused by the teachers' misunderstanding of

the question given. However, the teacher justified the student's answer stating that the numbers between $\frac{1}{3}$ and $\frac{1}{5}$ is $\frac{1}{4}$. Her reason was there is a 4 number between 3 and 5, thus there should be $\frac{1}{4}$ between $\frac{1}{3}$ and $\frac{1}{5}$. The following is a snippet of interviews between the researcher and the teachers.

- Researcher : One of your students answer like this (showing student's answer sheet who answer $\frac{1}{4}$). Is it correct?
- Teacher : It is correct. There is a 4 number between 3 and 5, thus there should be $\frac{1}{4}$ between $\frac{1}{3}$ and $\frac{1}{5}$.

The students' errors in comparing fractions were generated by students' perception that comparing fractions is similar to comparing integers. Students stated $\frac{3}{8} = \frac{9}{64}$ were incorrect as $\frac{3}{8}$ was not equal to $\frac{9}{64}$, they thought it should be $\frac{3}{8} < \frac{9}{64}$. Students compare $3 < 9$ dan $8 < 64$, accordingly $\frac{3}{8} < \frac{9}{64}$. This error was also found in mathematics teachers. The teacher was asked to illustrate two fractions so that the teacher was able to provide concrete explanations to the students. The teacher experienced misconceptions in illustrating the fraction that its numerator is greater than the denominator, for example in fractions $\frac{3}{4}$ and $\frac{4}{3}$. The teacher was able to illustrate fraction $\frac{3}{4}$ correctly, but she experienced misconceptions in illustrating fraction $\frac{4}{3}$. According to her, fraction $\frac{4}{3}$ was obtained by drawing $\frac{6}{3}$ first, then showing $\frac{3}{3}$ and then adding another shading which represents fraction $\frac{4}{3}$. The following figure shows the teacher's misconceptions in illustrating a fraction whose numerator is greater than its denominator.

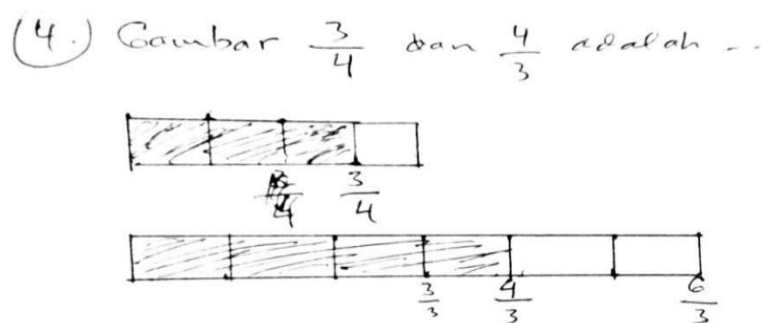


Figure 2: The teachers' misconceptions in illustrating fractions

The following is a snippet of interviews between the researcher and the teachers.

- Researcher : Would you like to explain the meaning of this and this illustration (while pointing at the pictures of fraction $\frac{3}{4}$ and $\frac{4}{3}$)?
- Teacher : For the picture of fraction $\frac{3}{4}$, I imagine it as a piece of wood divided by four, but only three parts are taken. That is what $\frac{3}{4}$ means. For the picture of fraction $\frac{4}{3}$, it this correct?
- Researcher : Well, according to you, is it correct?

Teacher : I think it should be $\frac{3}{3}$ if the shading is this (pointing to shading of $\frac{3}{3}$). If we want to illustrate $\frac{4}{3}$, we should add another shading.

In addition, in the students' exercise book, the researcher found the teacher's errors in comparing two fractions. The errors are shown in the students' illustration in comparing $\frac{6}{3}$ to $\frac{2}{3}$. However, the teacher justified the students' answer. Figure 3 below shows the teacher's error in comparing two fractions.

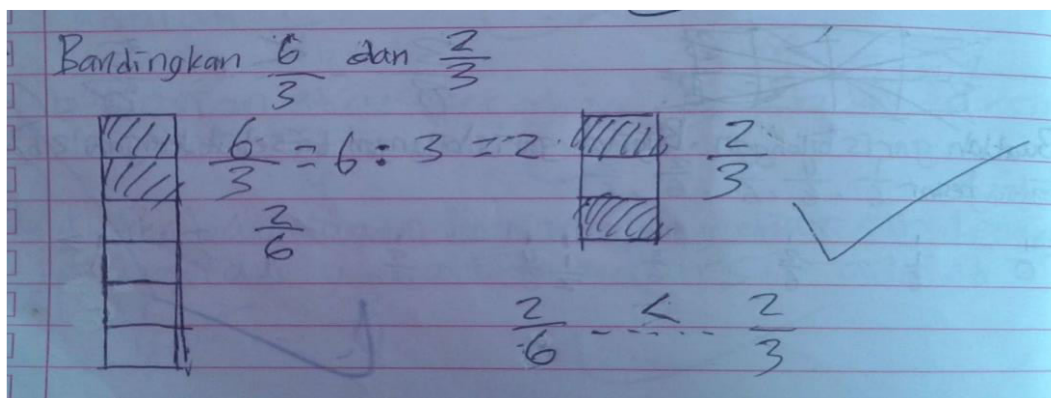


Figure 3: The teachers' error in comparing two fractions

Based on the findings above, the students experienced errors in comparing fractions by modeling them in the form of images. The same error was also experienced by the teacher in illustrating fractional numbers. When giving exercise to students, the teacher also performed errors in comparing two fractions. Therefore, the students' errors are potentially due to the teacher's errors in teaching comparing two fractions using images.

The students' errors in operating fractions were found in solving $\frac{4}{5} - \frac{3}{5}$. Students directly subtract the numerators (4-3) and the denominators (5-5), so the result is $\frac{4}{5} - \frac{3}{5} = \frac{1}{0}$. Given the fact, the researcher asked the math teacher about the results of fractions from $\frac{0}{0}$, $\frac{0}{5}$, and $\frac{5}{0}$. The teacher assumed that zero divided by zero equals to 1, which can be proven by making an example $\frac{7-7}{8-8} = \frac{0}{0} = 1$. The teacher answered that $\frac{0}{5}$ results zero, but she did not state the reason. While for the results of $\frac{5}{0}$, she answered that it is infinite as each number divided by zero results infinite number. The teacher's misconception in answering questions $\frac{0}{0}$, $\frac{0}{5}$, and $\frac{5}{0}$ is shown in Figure 4.

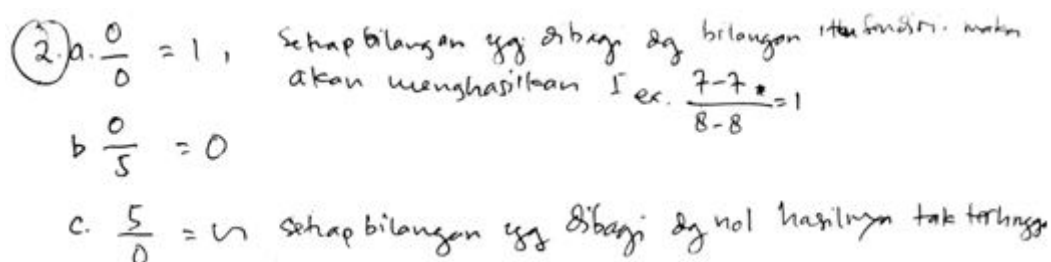


Figure 4: The teachers' misconception in dividing numbers by zero

The following is a snippet of interviews between the researcher and the teacher in dividing numbers by zero.

Researcher : *Can you give a more detailed explanation for number 2a?*

Teacher : *Ooo... Sure. For example $\frac{7}{7} = 1$, $\frac{8}{8} = 1$. It results the same when we make $\frac{7-7}{8-8} = \frac{0}{0} =$
1. Is that right?*

Researcher : *But it is different, the numerator is 7, while the denominator is 8.*

Teacher : *Yes, but after we subtract, the results is zero over zero.*

The teacher was able to express the concept of fractions correctly, but she could not associate it with the fraction $\frac{0}{0}$. Below is the excerpt of the interview.

Researcher : *Well, in your opinion, what is fractions?*

Teacher : *It is parts taken from the whole. For example, a pizza was cut into six parts or pieces, and when we take one part, it is one-sixth.*

Researcher : *Then, one-sixth is written as $\frac{1}{6}$, means 1 is the part we take, and six is the whole of the pizza, isn't it? In other words, we call the top part as numerator and the bottom part as the denominator. The next question is, is it possible for zero to be the denominator?*

Teacher : *(thinking). It seems correct. What is the correct answer? Is it infinite, similar to this 2c?*

Furthermore, the teacher was not able to describe the meaning of infinity. The following is the excerpt from the interview.

Researcher : *Why is it called as infinite? What is the meaning of infinite?*

Teacher : *Well... Maybe there are a lot of its values, countless.*

Based on the results of the interview above, it can be concluded that the teacher experienced misconceptions in dividing the number $\frac{0}{0}$. The teacher was wrong in making an example $\frac{7-7}{8-8} = \frac{0}{0} = 1$ because $\frac{0}{0}$ is undefined. Likewise, the correct answer for $\frac{5}{0}$ is undefined, instead of infinite. This was caused by the teacher's errors in defining the meaning of infinite and undefined.

Furthermore, the students' errors in understanding the story problems related to fractions were initiated by their errors in operating fractions. It was different from the misconceptions of the mathematics teachers in solving one of the 2016/2017 National Examination (UN) questions as follows: "Mother buys $7\frac{1}{2}$ kg sugar. 20% of the sugar is used to make cakes. The remaining unused sugar is" The teacher did not understand the meaning of "20% of". The teacher thought 20% can be simplified to $\frac{1}{5}$ so that the amount of sugar used by the mother can be solved by subtracting $7\frac{1}{2} - \frac{1}{5}$. The result was $7\frac{3}{10}$ kg. There was no $7\frac{3}{10}$ in the choices provided in the question, but the teacher was very confident in her answer and considered that the UN was wrong. The teacher's misconceptions in understanding story problems related to fractions is shown in figure 5 below.

$$\textcircled{5} \quad 7\frac{1}{2} - 20\% :$$

$$\frac{15}{2} - \frac{20}{100} = \frac{15}{2} - \frac{2}{10} = \frac{75}{10} - \frac{2}{10} = \frac{73}{10} = \left(7\frac{3}{10} \text{ kg}\right)$$

Figure 5: The teachers' misconceptions in understanding story problems related to fractions

The following are the excerpts of interviews between the researcher and the teachers related to the answer to the question.

Researcher : Well, could you explain your answer for question number 5?

Teacher : Sure, Mother has $7\frac{1}{2}$ kg of sugar. 20% is used. It decreases, doesn't it? Thus $7\frac{1}{2} - 20\%$. Let us see the answer options. There is no correct answer found. Is it possible for the UN question to be wrong?

Researcher : No, it is impossible; let's look to the question. What does the second sentence of this question mean?

Teacher : Umm... It is subtracted to 20%, right?

Researcher : 20% of the sugar mentioned before? (trying to emphasize the meaning of 20%)

Teacher : What do you mean?

Researcher : 20% was used, so, how many kgs is 20%?

Teacher : When 20% is simplified, it results $\frac{1}{5}$ kg.

Based on the results, it can be concluded that the teacher experienced misconceptions in answering story problems related to fractions. These misconceptions occur because of an algorithm error in operating fraction $7\frac{1}{2} \text{ kg} - 20\%$.

In addition, the researcher found the same errors when the teacher gave the exercises to the students. It was stated in the question that $\frac{1}{3}$ part of an aquarium is filled with water, then water was added again to the aquarium resulted $\frac{3}{4}$ parts of the aquarium is filled with water. The question is the part of the water added to the aquarium. However, the teacher misunderstood the question and justified the students' answer. Figure 6 shows the teachers' error in understanding story problems related to fractions.

3. Dik: air Aquarium $\frac{1}{3}$ air yang mau ditambah $= \frac{3}{4}$ (air dalam wadah)

Dit: Berapa air Aquarium + air yang di dalam wadah?

Jwb: $\frac{1}{3} + \frac{3}{4} = \frac{4+9}{12} = \frac{13}{12}$

Figure 6: The teachers' errors in understanding story problems

Based on the findings above, the students experienced errors in understanding story problems. The same errors were also experienced by the teachers, although the questions given were different. However, the teachers experienced errors in understanding the questions given to the students in the classroom. Therefore, the possibility of errors experienced by students was caused by the teachers in teaching story problems related to fractions.

5. Discussions

Misconceptions refer to misuses of rules or formulas, over generalizations, lack of generalizations or another conception of certain situation²³. Misconceptions shown on a topic identified the wrong interpretation of mathematical ideas as a result of personal experience or incomplete observation.

The teacher experienced misconceptions in ordering fractions indicated when the teacher answered the questions about numbers between $\frac{1}{3}$ and $\frac{1}{5}$ as well as $\frac{1}{6}$ and $\frac{1}{8}$. The teacher used a number line, but there was an error in determining the number. This error indicated a fractional misconception using a number line. In addition, the teacher only explained the number lines found in the school book. This fact shows that the teacher tended to use procedural methods, namely giving rules directly to be memorized, remembered, and applied by the students. There were no significant changes in the teaching methods applied by the teachers since their knowledge was still limited²⁴.

In addition, the teacher experienced misconceptions in illustrating the fraction whose numerator is greater than its denominator. Teachers are required to be able to provide illustrations in real or in the form of images; thus students can understand the subjects taught well. However, in reality, the teachers only gave examples of simple fractions, such as $\frac{1}{2}$, $\frac{1}{4}$, and so forth. Other findings suggest that 33% of teachers involved in a study of fractions did not use appropriate procedure and did not understand the concept proficiently in pretest, then reduced to 5% after posttest²⁵.

The misconception experienced by the teacher in dividing numbers by zero is shown in the answer of the question that asks the teacher to determine the results of $\frac{0}{0}$, $\frac{0}{5}$, and $\frac{5}{0}$. This is due to the lack of the teacher's knowledge about the basic concepts of fractions. It is indisputable that mathematics teachers need to possess knowledge of concepts from subject matters as it is impossible for teachers to teach mathematics without mastering knowledge of how to convey the mathematical concepts²⁶.

Furthermore, the teacher experienced misconceptions in understanding the story problems related to fractions. The error made was in understanding the meaning of "20% of" so that the results obtained were different from the correct answers. Another finding suggests that the misconceptions experienced by prospective teachers were in distinguishing expressions such as *of a*, *of one*, *of the*,

and of each¹². The story problems related to fractions was also given to 83 teachers, but only 49% understood and answered correctly²⁵.

Based on the misconceptions experienced by the teachers above, it can be concluded that the teachers' misconceptions occurred because the teachers did not master the subjects taught proficiently, completely, and correctly⁸. These misconceptions must be overcome promptly, since teachers should understand, master, and be skillful in using new resources in learning²⁷.

Basically, teachers must understand and comprehend the subjects taught to students, as teaching fraction requires attention, dedication, seriousness, perseverance, and professional competence²⁸. Teachers' professionalism is a set of competencies that must be possessed by teachers so that he/she can carry out his teaching tasks successfully²⁹. In addition, teachers must be able to explain the topics clearly, create interesting subjects, provide regular feedback, and provide assistance to students who do not understand the topics. This requires teachers' awareness to learn more about the mathematics topics being taught to students in order to encourage students' success in mathematics learning.

6. Conclusions

Based on the results and discussion, it can be concluded that the causes of misconceptions in the fractions experienced by the teachers are misconceptions of ordering fractions, misconceptions in illustrating the fractions that the numerator is greater than its denominator, misconception in dividing numbers by zero, and misconceptions in understanding story problems related to fractions.

The misconception in ordering fractions experienced by the teachers was caused by their misunderstanding the intention of the questions. The misconception in illustrating the fraction that the numerator is greater than its denominator was their misunderstanding the meaning of the fractions. The misconception in dividing numbers by zero was caused by assuming the smaller the denominator the greater the result. While the cause of the teachers' misconception in understanding the story problems related to fractions is their misunderstanding of the information given in the questions.

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